Theory and Experimental Results

To begin the investigation of the P, I, and PI control of the rotational speed for a DC brush motor, the parameters of this dc brush electro-mechanical motor will need to be identified. In particular, the motor back emf constant, Ke, will be found by inputting voltages in increments of 1V from 2-10V into the system while utilizing a BK power supply (+/- 15 Volts). This will allow the MUT to operate at different speeds and the output voltages are then recorded. These output voltages from the tachometer and the MUT are used to help find and verify the specification sheet’s tachometer sensitivity and torque constant.

A relationship between the angular velocity and output voltage from the MUT can be made to determine the motor voltage constant. Below, Equation 1 displays how the motor constant can be calculated.

(1)

Essentially, tachometer outputs are converted using the tachometer’s sensitivity given in the specification sheet to find the angular velocities. The slope is calculated from the data between the angular velocity measurements and the MUT voltage outputs to obtain the motor voltage constant. Next, the torque constant, Kt, can be calculated equating this value to the value of the motor voltage constant. However, a unit conversion will have to be made in order to get this value into units of torque. Once converted, the values just described will be used to obtain the system parameters.

The step response can be measured by powering the front motor (MUT) with a power op amp while implementing a function generator amplitude of +/- 8 volts so that the tachometer output will have a +/- 4 volt square wave output. During a step change, the outputs of both the tachometer output and motor input can be observed and analyzed to define the parameters. A system of equations from the MUT will be utilized to help determine the stall torque. Knowing that the back-drive motor is coupled with the MUT, the system of equations along with set up of the system is presented below.

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**Figure 1: Sketch of system used during data collection**

(2)

(3)

(4)

(5)

The stall torque, TL will occur when the angular velocity value is equal to zero due to a stall in a motor means there is no rotation occurring. Knowing that the angular velocity will be zero, Equation 6 will be simplified as shown below.

(6)

Data collected from the step change can be used to determine the *K* and τ of the motor. System gain will need to be calculated first which will result in the *Kmotor*value when divided by the tachometer sensitivity. A ratio of the final input motor voltage and final tachometer output will result in the system gain. Dividing this by the tachometer signal will result in units of KRPM/Volt. Equation 7 displays this relationship.

(7)

The time constant of the motor is found by observing the step input taken from the tachometer output voltage. By using the 63.2% method, the time constant will related to 63.2% of the tachometers max amplitude reading. These calculated parameters can be utilized to calculate the moment of inertia, J, and the damping coefficient, B, of the system. When the electrical time constant of the system is ignored, the system of equations can be rearranged as shown below in Equations 8 and 9.

(8)

(9)

All variables within the equations have been previously described and accounted for when determining the J and B values. The experimental values can then be compared to those given in the specification sheet for this dc motor to determine the accuracy.

A disturbance load of a 5 Ohm resistor is implemented on the open loop motor which will cause a decay rate in the voltage input of the tachometer once it is turned on. The decay time constant will be calculated in a similar manner as that of the motor. However, the amplitude of the open loop motor from the disturbance implementation will be multiplied by the 36.8% to determine the time constant.

**Results and Discussion**

After collecting data for the tachometer and MUT (See Table 1), the data could then be analyzed using the methods described earlier. The tachometer sensitivity of 3 V/KRPM was used to convert the Tach voltages to angular velocities.

**Table 1: Voltage input with resulting outputs.**

|  |  |  |  |
| --- | --- | --- | --- |
| Input, ei (Volts) | Tach, eo (Volts) | MUT, em (Volts) | Angular Velocity (KRPM) |
| 1.5 | 0.265 | 0.390 | 0.088 |
| 2 | 0.561 | 0.867 | 0.1870 |
| 3 | 1.14 | 1.756 | 0.38 |
| 4 | 1.72 | 2.661 | 0.573 |
| 5 | 2.38 | 3.85 | 0.793 |
| 6 | 2.994 | 4.85 | 0.998 |
| 7 | 3.579 | 5.806 | 1.193 |
| 8 | 4.195 | 6.784 | 1.398 |
| 9 | 4.794 | 7.768 | 1.598 |
| 10 | 5.405 | 8.751 | 1.802 |

From there, the MUT voltages were plotted against the angular velocities (See Figure 2). Using MATLAB’s ‘polyfit’ command, a line of best fit was created to help determine the slope of the data set. This slope is also equal to the value of the motor voltage constant, Ke. As a result the motor voltage constant equals 4.905 KRPM/V. The specification sheet lists an average constant of 4.88V within 200mV which means the experimental motor voltage constant falls within the range.

The torque constant, Kt was also accounted for and was assumed to be equal to Ke. However, the units had to be checked since the units for the torque constant will be in ozf-in/A. After checking the units, the motor voltage constant was multiplied by 141.6 to get the units into ozf-in/A. As a result, the Kt equals 6.63 oz-in/A while the specification lists a constant of 6.6 ozf-in/A. It can be said that accurate data measurements were taken due to the precision of the constants calculated for the motor voltage and the torque.

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**Figure 2: Displays MUT Voltage vs Angular Velocity**

The derived system equations for the motor under test can be found above in Equations 2-4. Using these equations will help determine the torque of the motor when there is zero velocity. This can also be stated as the stall torque and using Equation 6 from above will with the known 6V input, torque constant, and the motor terminal resistance of 3.6 ohms, a stall torque of 11.05 ozf-in is calculated. The terminal resistance of the motor ranges from 3.6-4.9 Ohms. However, the smallest resistance is desired so the stall torque can be calculated since there will be zero velocity. Using the data collected during the step change (See Figure 3), K­motor can be calculated using Equation 7. This results in a value of 0.177 KRPM/V. This value will be needed to determine the damping coefficient as well as the moment of inertia. The time constant was determined by using the 63.2% method of the amplitude from the tachometer. Since the tachometer ranges from the -4V to 4V, the amplitude had to be subtracted by 4 to account for the voltage difference. As a result, the speed of the response came had a resulting time constant value of 0.0089 seconds.

The moment of inertia and damping coefficient values were determined when the electrical time constant was ignored. Equations 8 and 9 were used to solve for these parameters. B was determined to be 0.578 oz-in/KRPM and could then be plugged into Equation 9 to solve for J. The moment of inertia was calculated to be 0.00035 oz-in-sec2. The table below shows the experimental parameters compared to the theoretical.

**Table 2: Moment of Inertia and Damping Coefficient values**

|  |  |  |
| --- | --- | --- |
|  | Moment of Inertia | Damping Coefficient |
| Theoretical | 0.0004 oz-in-sec^2 | 0.578 oz-in/KRPM |
| Experimental | 0.00035 oz-in-sec^2 | 0.2 oz-in/KRPM |

The moment of inertia values matches up very nicely to one another. However, the damping coefficient seems to be a little over double the specification sheets value. The reason behind this may be due to ignoring the electrical time constant.

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**Figure 3: Tachometer Output and Motor Input due to step change**

The figure below (Figure 4) represents the decay rate from the motor when disturbance from the 5 Ohm resistor was used. Using the decay time constant, the time constant of the system when subjected to a 5 Ohm load resistor is 0.0082 seconds. This compares very well to the time constant of the open loop system with disturbance. The reason for any difference could be due to the noise in the decay rate. An aggressive smoothing function was used to try and eliminate noise but still ended up having a significant amount. The settling time was also calculated by multiplying the decay time constant by 4 since that is ideally the time it will take for the response to settle. This resulted in a time of 0.033 seconds which when looking at Figure 3, the response has settled by this time.

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**Figure 4: Represents decay rate from the disturbance load of 5 Ohms**